Mrs. Robertson

Calculus II
Test 4

Solutions Test 4 Review

1. Determine whether the sequence converges or diverges. If it converges, state the limit.

a. $a_n = \frac{3+7n-5n^3}{2n^3+3n^2-1}$

$$\lim_{n\to\infty} \frac{3+7n-5n^3}{2n^3+3n^2-1} = \lim_{n\to\infty} \frac{\frac{3}{n^3}+\frac{7}{n^2}-5}{2+\frac{3}{n}-\frac{1}{n^3}} = \frac{-5}{2}$$

$\boxed{\text{converges to } \frac{-5}{2}}$

b. $a_n = \frac{n}{\sqrt{9n^2+1}}$

$$\lim_{n\to\infty} \frac{n}{\sqrt{9n^2+1}} = \frac{\frac{n}{n}}{\sqrt{\frac{9n^2}{n^2}+\frac{1}{n^2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

$(n = \sqrt{n^2})$

$\boxed{\text{converges to } 1/3}$

2. Find the exact value for the series $\sum_{k=1}^{\infty} \frac{6}{(k+2)(k+3)}$

$$\frac{6}{(K+2)(K+3)} = \frac{A}{K+2} + \frac{B}{K+3}$$

$6 = A(K+3) + B(K+2)$

$6 = AK + 3A + BK + 2B$

$0 = AK + BK$ $\quad$ $6 = 3A + 2B$

$0 = A + B$ $\quad$ $6 = 3(-B) + 2B$

or $\quad$ $6 = -B$

$A = -B$ $\quad$ $-6 = B$

$6 = A$

$$\sum_{k=1}^{\infty} \left(\frac{6}{K+2} - \frac{6}{K+3}\right) =$$

$$\left(\frac{6}{3} - \cancel{\frac{6}{4}}\right) + \left(\cancel{\frac{6}{4}} - \cancel{\frac{6}{5}}\right) + \left(\cancel{\frac{6}{5}} - \cancel{\frac{6}{6}}\right) + \dots +$$

2

$$\cancel{\frac{6}{K+2}} - \frac{6}{K+3}$$

$$= 2 - \lim_{n\to\infty} \frac{6}{K+3} \to 0$$

$= \boxed{2}$

3. Determine whether the following geometric series converge or diverge. If they converge, find the exact value of the sum. Clearly state a and r.

a. $\sum_{k=0}^{\infty}\left(-\frac{2}{3}\right)^k$

$|r| = |-2/3| < 1$ so... converges

$\boxed{a = 1 \quad r = -2/3}$

$\text{Sum} = \frac{a}{1-r} \Rightarrow \frac{1}{1-(-2/3)} = \frac{1}{\frac{5}{3}} = \boxed{\frac{3}{5}}$

b. $\sum_{k=1}^{\infty} 12(0.01)^k$

$\text{Sum} = \frac{0.12}{1-0.01} = \frac{.12}{.99} = \boxed{\frac{4}{33}}$

$\boxed{a = 0.12 \quad r = 0.01}$ < 1 so... converges

c. $\sum_{k=1}^{\infty} \frac{5^k}{2^{k+1}} = \frac{5}{4} + \frac{28}{8} + \frac{125}{16}$

$r = \frac{\frac{28}{8}}{\frac{5}{4}} = \frac{28}{8} \cdot \frac{4}{5} = \frac{5}{2}$

$\boxed{a = \frac{5}{4} \quad r = \frac{5}{2}}$ > 1 so... diverges

4. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. Indicate the test used.

11.6 ~~a.~~ $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k^2+1}}$

11.6 ~~b.~~ $\sum_{k=1}^{\infty} (-1)^k \frac{k}{e^k}$

5. **Determine whether the following series converge or diverge. Clearly state either the test for convergence used or the type of series. Also clearly state the reasoning for the conclusion.**

omit

11.6

X a. $\sum_{k=1}^{\infty}\left(\frac{7k+1}{9k+3}\right)^k$

X b. $\sum_{k=1}^{\infty}\frac{5^k}{(k+1)!}$

✓ c. $\sum_{k=1}^{\infty}\frac{k^2}{5k^2-3k}$ $\lim_{K\to\infty}\frac{K^2}{5K^2-3K}=\lim_{K\to\infty}\frac{1}{5-\frac{3}{K}}=\frac{1}{5}\neq 0$

divergence test – the series diverges

✓ d. $\sum_{k=1}^{\infty} \frac{k^2+1}{k^3-2}$ $\frac{K^2+1}{K^3-2} > \frac{1}{K}$ Since $\frac{1}{K}$ is a divergent series

then $\frac{K^2+1}{K^3-2}$ is divergent by the comparison

test.

✓ e. $\sum_{k=1}^{\infty} 5k^{-2/3}$ $= \sum_{k=1}^{\infty} \frac{5}{K^{2/3}}$ p-series with $p = 2/3 < 1$

$\therefore$ diverges

✗ f. $\sum_{k=1}^{\infty} \frac{\sin k}{2^k}$